PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A | MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Pollution Problems in Relation to the Thames Barrier

M. J. Barrett and B. M. Mollowney

Phil. Trans. R. Soc. Lond. A 1972 **272**, 213-221
doi: 10.1098/rsta.1972.0047

Phil. Trans. R. Soc. Lond. A. **272**, 213–221 (1972) [213]
Printed in Great Britain

Pollution problems in relation to the Thames barrier

By M. J. Barrett and B. M. Mollowney
Water Pollution Research Laboratory of the Department of the Environment, Stevenage, Herts

Introduction

Although the barrier, which is to be constructed about 15 km seawards of London Bridge, need only be closed to exclude exceptionally high tides to fulfil its primary function as a flood-prevention device, its design is such that it could be operated as a tide-control structure, closing at a particular stage during each ebb and opening at a similar stage on the succeeding flood, unless a storm surge is forecast. This paper outlines a theoretical study, undertaken at the request of the Greater London Council, to assess the effect on the condition of the water of the Thames Estuary of operating the barrier on a regular basis in such a way as to prevent water levels landwards of it falling below Newlyn Datum.

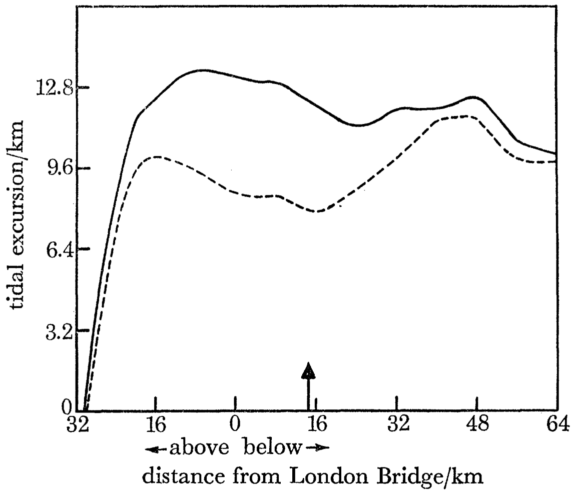


Figure 1. Tidal excursion with (---) and without (—) tide control. Arrow shows position of barrier.

Methods for predicting the effect of polluting discharges on the Thames Estuary, which is one in which the tidal excursion is short in comparison with its length, and in which vertical salinity gradients are small, have been published in a comprehensive report (Department of Scientific and Industrial Research 1964). The successful outcome of this work hinged on the development of a mathematical model for describing the influence of various factors on the distribution of dissolved oxygen in the estuary. Although the present study owes much to this earlier work, the quantized model (Preddy 1954) previously adopted for describing the way in which effluents would become dispersed as a result of tidal oscillation and displacement by the freshwater flow, was not considered suitable in that only changes occurring from one tidal cycle to the next were considered.

Studies by the Hydraulics Research Station (Greater London Council 1969) on a large-scale physical model indicate that tide control would bring about fundamental changes in the estuary. The tidal range landwards of the barrier would, of course, be reduced by the removal of the lower half of the tide curve, and in a volume of the upper estuary formerly emptying between half-ebb and low water, current velocities would fall almost to zero. There would be a consequent reduction in tidal range and hence in the tidal excursion – the distance that a molecule of water would travel along the estuary from one slack water to the next in the absence of displacement by freshwater (figure 1). There would also be changes in salinity. The overall variation at a given position would be reduced and, for the first few tides after tide control was imposed, the upstream limit of the salt water would be displaced gradually until it became established some 4 km farther seawards. When tide control was discontinued the original salinity distribution would quickly be re-established. To assess the implication of these changes a one-dimensional time-dependent numerical model, which encompasses the ebb and flow of the tide, has been developed.

MATHEMATICAL MODEL

Representation of tidal movement

For the purposes of the model the estuary is considered as a series of uniformly mixed segments moving upstream and downstream with the tide, such that the volume of water landwards of a given segment boundary remains constant. Let X_0^0, X_i^0, X_n^0 be the positions of the segments boundaries at time $t = 0$, taken to be high water at Southend. The position, X_i^t, of a particular boundary at various times during the tidal cycle is calculated numerically by a procedure similar to that illustrated diagrammatically in figure 2. In the upper part the cumulative volume between the tidal limit and particular points in the estuary is shown for different values of time, t. A line of constant volume, say $V = V_i$, defines discrete points X_i^0, X_i^1, X_i^2 which are plotted

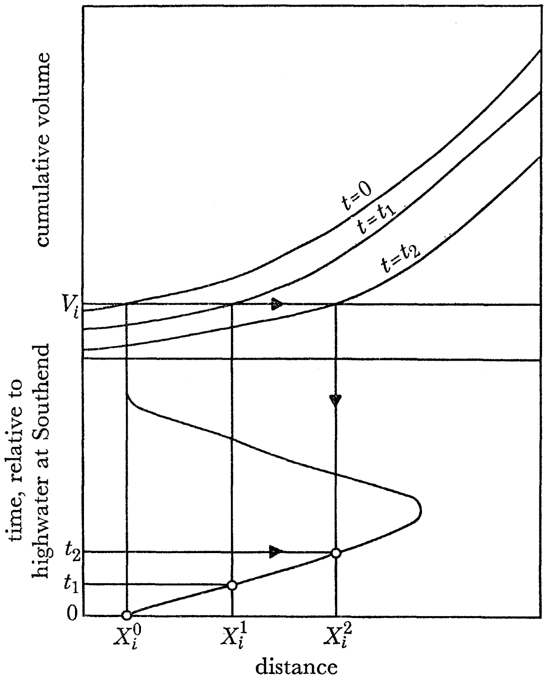


FIGURE 2. Diagrammatic representation of the movement of water during a tidal cycle. Symbols defined in text.

against time as shown in the lower half of the figure. The resultant curve is associated with the volume V_i, which is the volume upstream of X_i^0 at $t = 0$. The curves shown in figure 3 are those calculated for the tidal limit at Teddington Weir (30.5 km upstream of London Bridge) and at intervals of 6.4 km throughout the estuary, using the areas of 19 cross-sections surveyed by the

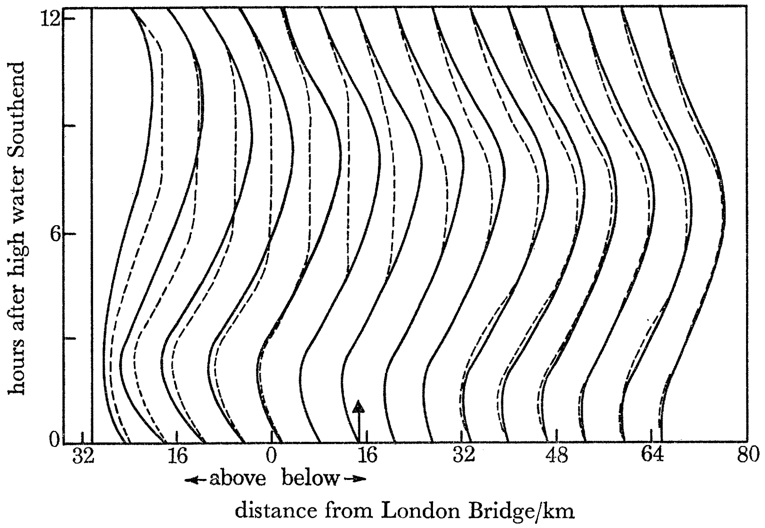


FIGURE 3. Curves representing movement of water during a mean tidal cycle, - - -, with control; ——, without control.

Port of London Authority in 1948 and variations in tidal heights recorded at 13 positions in the hydraulic model operated with and without tide control. It will be appreciated that although segment volumes are constant, their lengths, $X_i - X_{i-1}$, and surface areas, SA_i, vary throughout the tidal cycle. To satisfy continuity of volume there is a seaward flow through each segment boundary equal to the net freshwater flow entering from all sources landwards of it.

Longitudinal dispersion

The Thames is a slightly stratified estuary and the variations in concentration of dissolved substances over the depth and between the centre and sides are not large. Consequently interpretation of the distribution of pollutants can be treated as a problem in one dimension, the average concentration of a substance over the cross-section being expressed as a function of distance along the length of the estuary. But variations in concentration along the cross-section when combined with the variations in velocity give rise to a net longitudinal mass transport. This is called longitudinal dispersion and is simulated in the model by assuming a continuous exchange of water between adjacent segments by equal and opposite flows as indicated in figure 4 by F_i. This simulation is essentially a finite difference approximation to the model where longitudinal dispersion is assumed to be a diffusion process with

$$F_i = \frac{2D_i A_i}{(X_{i+1} - X_i)}, \tag{1}$$

where D_i is the diffusion coefficient and A_i is the cross-sectional area at X_i. The magnitude of this exchange flow was calculated from the equilibrium salinity distribution by assuming a balance at any instant between the mass of salt transported by the net freshwater flow and that transported by the exchange flow. Thus if S_i is the equilibrium salinity at any instant in the segment X_{i-1} to X_i, F_i is calculated from the equation:

$$F_i(S_{i+1}-S_i) = Q_i S_i, \tag{2}$$

where Q_i is the freshwater flow at X_i. The mixing exchange in the upper regions, where salinity changes were too small to detect, has not been measured, but has been assumed to be of an order similar to that found in the Potomac Estuary by Hetling & O'Connell (1966) from an analysis of a continuous dye experiment. Upstream of the barrier no longitudinal mixing was assumed to occur during the period of closure. Because of the assumption that the segments are uniformly mixed some numerical mixing in this region is unavoidable but this can be made quite small by having short segments.

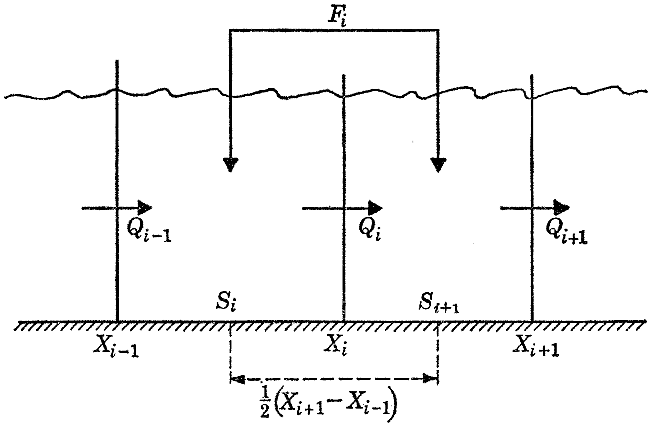


FIGURE 4. Longitudinal dispersion and freshwater flow as represented in the mathematical model.

Calculation of the distribution of a substance

The distribution of a substance is calculated by considering the changes in mass occurring in each segment during successive intervals of a tidal cycle. Thus if C_i is the concentration in the segment X_{i-1} to X_i, the mass balance in the ith segment is given by

$$V_i\,\mathrm{d}C_i/\mathrm{d}t = Q_{i-1}C_{i-1}-Q_iC_i+F_{i-1}\,(C_{i-1}-C_i)+F_i(C_{i+1}-C_i)-K_iV_iC_i+M_i, \tag{3}$$

where Q_i is the freshwater flow at X_i, F_i is the exchange flow at X_i, K_i is the rate of decay of the substance, M_i is the mass rate at which the substance is added from the surface, sides, and bottom of the estuary to the ith segment, and t is the time.

If there are n segments covering the estuary there will be n equations similar to equation (3) for $i = 1, 2, 3, \ldots, n$. This set of differential equations was solved at discrete time intervals using the Crank–Nicholson approximation. This consists of integrating equation (3) from $t = (m-1)\Delta t$ to $t = m\Delta t$, where m is an integer taking values 1, 2, 3, . . ., and Δt is the time step, and approximating the integral on the right-hand side by the trapezoidal rule. Equation (3) becomes

$$\begin{aligned} V_i(C_i^m-C_i^{m-1}) = {} & \tfrac{1}{2}\Delta t\{Q_{i-1}^m C_{i-1}^m - Q_i^m C_i^m + F_{i-1}^m(C_{i-1}^m - C_i^m) \\ & + F_i^m(C_{i+1}^m - C_i^m) - K_i V_i C_i^m\} \\ & + \tfrac{1}{2}\Delta t\{Q_{i-1}^m C_{i-1}^{m-1} - Q_i^m C_i^{m-1} + F_{i-1}^m(C_{i-1}^{m-1} - C_i^{m-1}) \\ & + F_i^m(C_{i+1}^{m-1} - C_i^{m-1}) - K_i V_i C_i^{m-1}\} + M_{i,\,\mathrm{add}}^m \end{aligned} \tag{4}$$

where C_i^m = concentration in the ith segment at $t = m\Delta t$, F_i^m and Q_i^m are the average values at

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

X_i during the time step $(m-1)\Delta t$ to $m\Delta t$, and $M^m_{i,\,\mathrm{add}}$ is the total mass of substance added to the segment during the time step. Equation (4) can be rearranged to give an equation of the form

$$\alpha_i C^m_{i-1} + \beta_i C^m_i + \gamma_i C^m_{i+1} = \delta_i(C^{m-1}_{i-1}, C^{m-1}_i, C^{m-1}_{i+1}). \tag{5}$$

The concentration of a substance is given arbitrary initial values and the set of algebraic equations (5) is solved using the Gauss–Seidel iterative method. The values obtained after one time step are then used as initial values for the next time step. The calculation is carried on until the solution shows a periodic variation with no trend from one tidal cycle to the next. This is taken to be the 'equilibrium' situation that would develop under steady conditions.

Calculations of the effect of tide control

Salinity

Calculated variations in salinity during a tidal cycle without the barrier, at a number of positions from 56.3 to 0.3 km below London Bridge, are compared in figure 5 with observations made on the hydraulic model operated so as to simulate a tide with a range of 4.4 m at Southend and freshwater flow of 34 m³/s at Teddington Weir. Figure 6 gives the corresponding comparison with the barrier in operation.

Although slack-water salinity values are not exactly reproduced, the overall agreement between the mathematical and physical models is satisfactory. This mutual confirmation suggests that either model simulates the effects of tidal motions in the estuary with a fair degree of accuracy.

Dissolved oxygen and associated substances

The concentrations of organic carbon, organic nitrogen, ammonia and oxidized nitrogen have to be calculated simultaneously with that of dissolved oxygen. The assumptions relating to the oxidation of organic matter and ammonia and to the reduction of oxidized nitrogen were those previously used (Department of Scientific and Industrial Research 1964), namely that when the dissolved oxygen is plentiful:

(1) The rates of oxidation of carbon compounds are proportional to their concentrations.

(2) The organic nitrogen is converted to ammonia at the same rate as the carbon is oxidized.

(3) The rate of oxidation of ammonia is proportional to its concentration.

(4) Oxidized nitrogen is destroyed at a very slow rate proportional to its concentration.

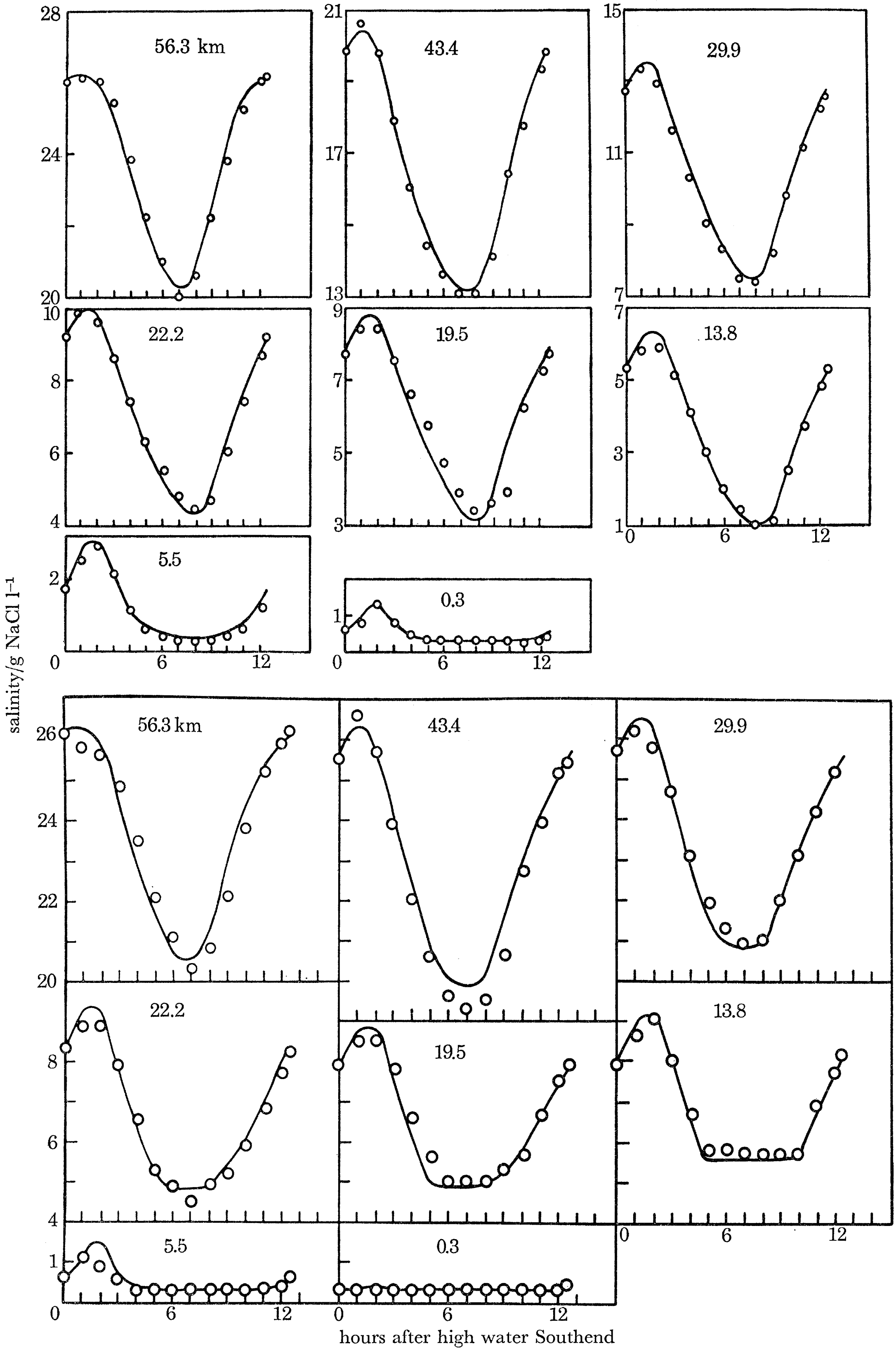
However, when the dissolved oxygen becomes less than 5 % of the air saturation value assumptions 3 and 4 are replaced by the following:

(5) The rate of oxidation of ammonia is retarded, and if in spite of this assumption the dissolved oxygen still falls below 5 % of saturation, the rate of oxidation of ammonia is taken as zero and oxidized nitrogen is reduced to molecular nitrogen at a rate sufficient to maintain the oxygen level at 5 % of saturation.

Furthermore, as before, it is assumed that the rate of solution of oxygen from the atmosphere is proportional to the oxygen saturation deficit. When this assumption is applied to the ith segment taking into account this process alone

$$V_i \mathrm{d}C_i/\mathrm{d}t = f_i S A_i\,(C_{si} - C_i), \tag{6}$$

where C_i is the concentration of dissolved oxygen, C_{si} is the air saturation value of dissolved



FIGURES 5, 6. Salinity variations during a tidal cycle at given distances below London Bridge. Figure 5 (top) shows variations without operation of the barrier and figure 6 (bottom) with the barrier in operation. ○, observed data from model; —, calculated results. Equivalent freshwater flow, 34 m³/s at Teddington; tidal range 4.4 m at Southend.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

oxygen, taking temperature and salinity into account, SA_i is the surface area, V_i is the volume, and f_i is the re-aeration coefficient.

The effect of the proposed form of tide control on the distribution of dissolved oxygen and of ammoniacal and oxidized nitrogen in the estuary at various stages during the tidal cycle is illustrated in figure 7. Estimates of the loads are based on data provided by the G.L.C. for the major sewage works, assuming completion of extensions to the Beckton works. Estimates of loads from tributaries are based on information in the Annual Report of the Scientific Branch of the G.L.C. 1968–9. There is a slight deterioration in the condition of the estuary resulting from tide control, except for a small area in the region of low dissolved-oxygen content where there is some improvement.

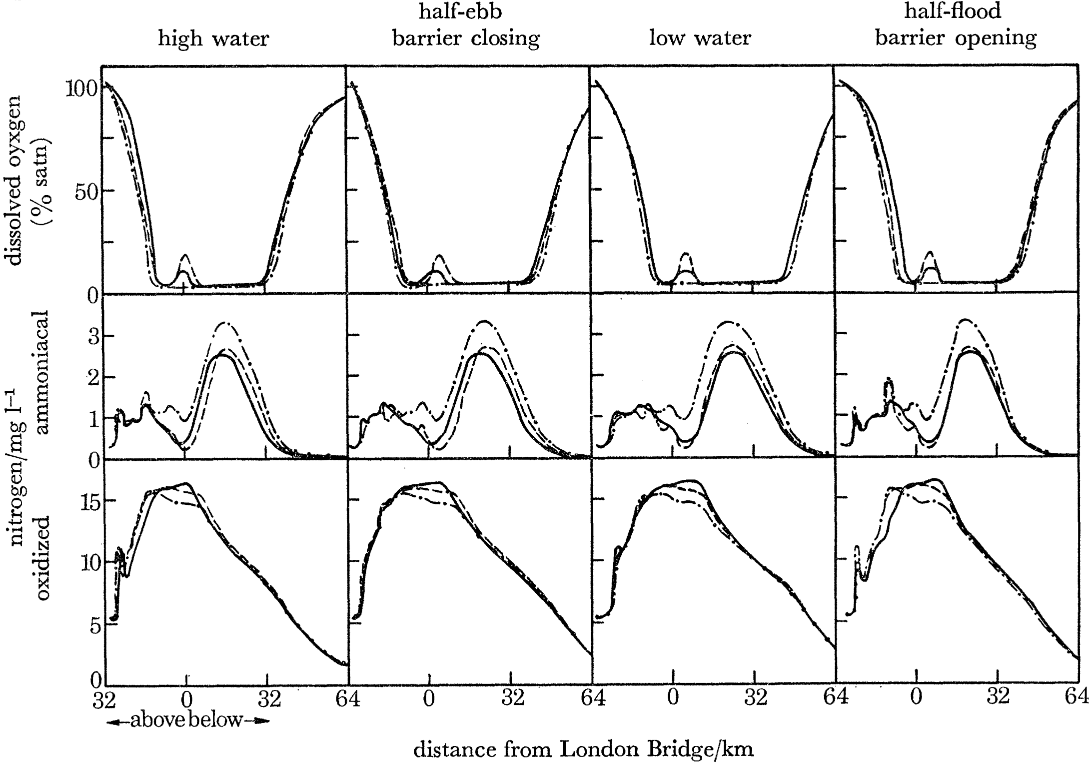


FIGURE 7. Predicted effect of half-tide control on distributions of dissolved oxygen and of ammoniacal and oxidized nitrogen in Thames Estuary. Temperature, 22 °C; flow 15.8 m^3/s at Teddington. ——, without control; – – –, with control; –·–·–, with control (see text).

The average value of the re-aeration coefficient, f, is known fairly well, but not its variation through the length of the estuary. Therefore a case was calculated (chain line in figure 7) using what is regarded as a lower limit for f of 2.5 cm/h upstream of the barrier during the period of closure and 5.5 cm/h otherwise. One feature is the stretching of the zone of low dissolved-oxygen content during the tidal cycle, which is seen by comparing the distribution of dissolved oxygen at half-tide with the barrier closing and that with it opening.

Temperature predictions

It is convenient to consider the temperature of the estuary at any point to be composed of two parts – the basic temperature which is the temperature that would exist in the absence of artificial heating and the temperature rise which is that part of the temperature attributable to cooling water discharges. Since tide control should have no effect on the basic temperature, predictions of the effect of the barrier have been made in terms of the temperature rise rather than the actual temperature. Using an approach similar to that for oxygen exchange it was assumed that the rate of loss of heat to the air from the ith segment is proportional to the surface area SA_i and the temperature rise T_i. Thus, taking into account this process only,

$$V_i \mathrm{d}T_i/\mathrm{d}t = -gSA_i\,T_i, \qquad (7)$$

where g is the heat-exchange coefficient (Gameson, Hall & Preddy 1957). The predicted effect of the proposed form of tide control on the distribution of temperature rise for a freshwater flow of 15.8 m³/s and a value of the heat-exchange coefficient of 3.7 cm/h is illustrated in figure 8, using information supplied by the Central Electricity Generating Board for the maximum rate of heat rejection by generating stations in October 1980.

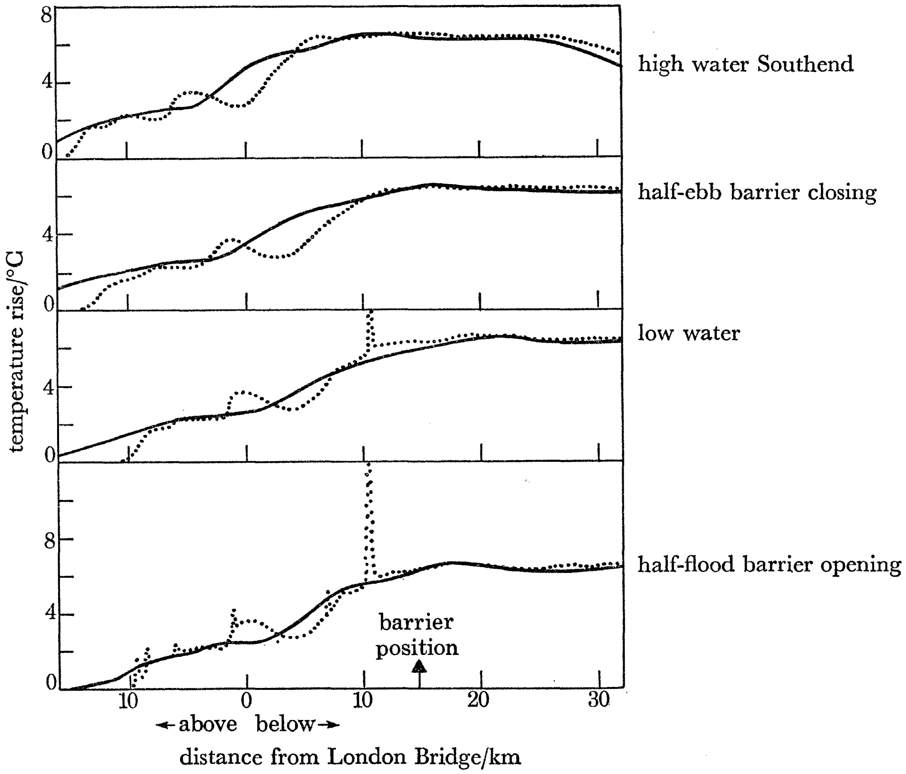


Figure 8. Predicted effect of half-tide control on temperature of Thames Estuary. Freshwater flow, 15.8 m³/s at Teddington; tidal range 4.4 m at Southend. ·····, with control; —, without control.

Although peaks of temperature occur in the vicinity of outfalls landwards of the barrier while the gate is closed, these quickly disappear when the gate is opened and the tidal flow is restored (figure 9). The model takes no account of lateral or vertical variations in temperature and assumes that the heat inputs are uniformly mixed across the estuary, and it is therefore to be expected that at some points in a cross-section close to an outfall actual temperature rises would be higher, and at other points further from the outfall they would be lower than those shown by the curves.

That the temperature peaks are proportional to the freshwater flow can be seen from figure 10 where the temperature rise at Brunswick Wharf (10.6 km below London Bridge) for a flow of

15.8 m³/s is compared with that for a flow of 34 m³/s. These peaks could be reduced by arranging for a flow across the barrier when it is closed. For example, for a flow of 15.8 m³/s at Teddington and with 31.6 m³/s flowing across the barrier the peak temperature rise would be reduced by 2.8 °C and at the same time there would be an average drop in water level of only 3.3 cm upstream of the barrier.

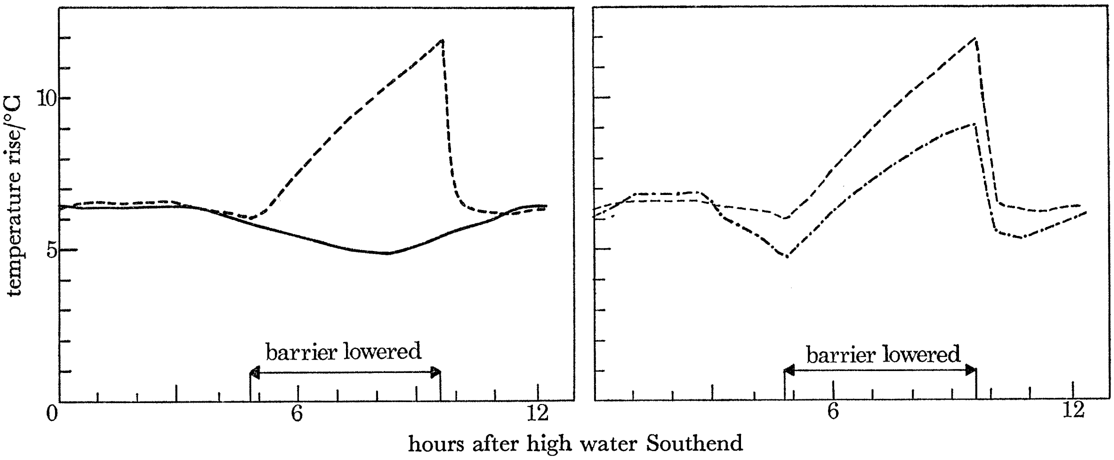


Figure 9

Figure 10

Figures 9, 10. Predicted variations in temperature rise off Brunswick Wharf, 10.6 km below London Bridge during a tidal cycle

Figure 9 (left): ---, with control; —, without control, for a freshwater flow of 15.8 m³/s at Teddington; a tidal range of 4.4 m; and heat exchange coefficient 3.7 cm/h.

Figure 10 (right): for two freshwater flows at Teddington of ----, 15.8 m³/s and -·-·-·-·-·-34 m³/s, with the barrier in operation.

References (Barrett & Mollowney)

Department of Scientific and Industrial Research 1964 Effects of polluting discharges on the Thames Estuary. *Water Pollution Research Technical Paper* No. 11. London: H.M. Stationery Office.
Gameson, A. L. H., Hall, H. & Preddy, W. S. 1957 *Engineer* **204**, 816, 850, 893.
Greater London Council 1969 *Thames flood prevention*, p. 78. First Report of Study.
Hetling, L. J. & O'Connell, R. L. 1966 *Water Resources Res.* **2**, 825.
Preddy, W. S. 1954 *J. mar. biol. Ass. U.K.* **33**, 645.